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Canonical approach to Courant brackets for D-branes

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Abstract

We present an extension of the Courant bracket to the ones for Dp-branes by analyzing Hamiltonians and local superalgebras. Contrast to the basis of the bracket for a fundamental string which consists of the momentum and the winding modes, the ones for Dp-branes contain higher rank R-R coupling tensors. We show that the R-R gauge transformation rules are obtained by these Courant brackets for Dp-branes where the Dirac-Born-Infeld gauge field and the “two-vierbein field” play an essential role. Canonical analysis of the worldvolume theories naturally gives the basis of the brackets and the target space backgrounds keeping T-duality manifest at least for NS-NS sector. In a D3-brane analysis S-duality is manifest as a symmetry of interchanging the NS-NS coupling and the R-R coupling.

1 Introduction and summary

In string theories the general coordinate transformation symmetry is enlarged to the gauge symmetry for the Kalb-Ramond field in addition to the gravitational field. This is an inevitable feature from T-duality of string theories which mixes the gravitational field and the Kalb-Ramond field. T-duality has a long history of the research [1, 2, 3]. Along the study of T-duality as a target space duality Siegel wrote down the gauge transformation rule of the gravitational field and the Kalb-Ramond field, G_{mn} and B_{mn} , in a T-duality covariant way [4, 5, 6]. Hitchin introduced the generalized geometry with the Courant bracket which gives this gauge transformation involving B_{mn} field [7]. Hull introduced the doubled formalism with manifest T-duality to flux compactifications by introducing non-geometry [8]. Hull and Zwiebach used the closed string field theory to construct the double field theory defined by the C-bracket which is reduced to the Courant bracket [9].

In bosonic string theory both T-duality and the gauge symmetry of G_{mn} and B_{mn} are governed by $O(d,d)$ symmetry consistently. The momentum and the winding modes of a string are the building block of $O(d,d)$ vector. On the other hand in type II superstring theory there are R-R gauge fields. They couple to D-branes whose charges are transformed as a spinor under $SO(d,d)$. T-duality interchanges IIA D-branes and IIB D-branes. Furthermore the IIB theory includes S-duality. Then T-duality is enlarged to “U-duality” [10, 11]. Corresponding to this enlargement the gauge symmetry involving R-R gauge fields should be enlarged. M theory is a powerful theory to explore U-duality and the enlarged gauge symmetry cooperative to U-duality [12, 13]. There is also an approach from the supergravity theory [14, 15]. In this paper we focus on a D-brane extension of the gauge transformation given by a new type of Courant bracket, leaving the U-duality problem for D-branes. We clarify the background field dependence of Hamiltonians for D-branes, and reveal differences between the fundamental string case and D-brane cases.

We take a canonical approach of worldvolume theories to explore the enlarged gauge symmetry for D-branes. For the fundamental string the momentum and the winding modes construct an $O(d,d)$ vector $Z_M = (p_m, \partial_\sigma x^m)$. The canonical Hamiltonian \mathcal{H}_\perp and the σ -diffeomorphism constraint \mathcal{H}_\parallel are expressed in terms of the basis as $\mathcal{H}_\perp = Z_M \mathcal{M}^{MN} Z_N$ and $\mathcal{H}_\parallel = Z_M \tilde{\eta}^{MN} Z_N$ with an off-diagonal $O(d,d)$ invariant metric $\tilde{\eta}^{MN}$. The target space background fields are included only in \mathcal{M}^{MN} . This expression has manifest T-duality symmetry. On the other hand the gauge symmetry is generated by Z_M . The canonical bracket between Z_M 's makes a closed algebra including a stringy anomalous term. This anomalous term is proportional to the $O(d,d)$ invariant metric. The Courant bracket is obtained by reading off from the regular coefficient of this canonical algebra [4, 5, 16]. The gauge transformation of G_{mn} and B_{mn} is given by the Courant bracket between the “two-vierbein field” and a gauge parameter, where the “two-vierbein field” is an $O(d,d)$ vector representation of G_{mn} .

and B_{mn} [4]. This is an ideal $O(d,d)$ vector for both T-duality and the gauge symmetry.

Now the problem is an extension of this analysis to Dp -branes. There are proposals of extensions of the Courant bracket for p -branes in [12, 14]. When we try to extend it in canonical approach we face to two crucial differences from string: 1. A Dp -brane has p worldvolume spatial directions, so replacing $\partial_\sigma x^m$ by $\partial_i x^m$ with $i = 1, \dots, p$ does not work out straightforwardly. 2. R-R gauge fields are higher rank tensors whose treatment in the framework of the Courant bracket is unknown. These problems are partially resolved by the help of the Dirac-Born-Infeld (DBI) $U(1)$ gauge field and the two-vierbein field. For the first problem the cotangent vector corresponding to the winding mode is constructed as $E^i \partial_i x^m$ where E^i is DBI gauge field strength [17, 18, 19]. For the second problem we found that the basis of the Courant bracket for Dp -brane consists of the higher rank tensors. The R-R gauge fields build a vector in this enlarged space by contracting with the two-vierbein field. Then we show that the R-R gauge transformation rules are generated by our Courant brackets for Dp -branes which contains Chern-Simons terms. In the reference [14] the exceptional Courant bracket contains Chern-Simons terms. In our approach the Chern-Simons terms are obtained from the canonical commutator between DBI gauge fields.

This paper is organized as follows: In section 2 we analyze the gauge generator algebra and the Courant bracket for a fundamental (F) string, and we extend it to the one for a D3-brane from their local superalgebras in flat space. Several extensions of the Courant bracket to the one involving p -form were introduced in [20]. In section 3 background fields are taken into account in this formulation for a F-string. We also show that a string on a group manifold such as AdS space has the same structure as the Courant bracket. The similar bracket was introduced in [6, 21]. For a D-string we demonstrate how the R-R coupling causes differences from the F-string case, and we present an extension of the Courant bracket to reproduce the gauge transformation of the R-R gauge field. In section 4 the above analysis is extended for IIA Dp -branes and IIB Dp -branes. Obtained basis of Courant brackets and background matrices are subsets of whole U-duality. In order to construct a U-duality manifest theory these subsets will be combined in some sense.

There are many interesting researches on this subject; generalized geometry to flux compactifications in physics [22], recent work on double field theory [23] and doubled formalism and D-branes [24, 25].

2 Flat background

D-brane is solitonic excitation in type II superstring theories and the BPS condition is given by the $N=2$ supersymmetry. The BPS mass is determined from the global supersymmetry

algebra, while local information such as the Virasoro condition is determined from the local superalgebra. The local superalgebra is written in terms of supercovariant derivatives, $(d_{A\alpha}, p_m \pm \partial_\sigma x^m + \dots)$ in flat space with spacetime vector index m , spinor index α , and $N=2$ supersymmetry index A . We can read off a complete set of the bosonic basis of the R-R coupling from the right hand side of the local superalgebra, $\{d_{A\alpha}, d_{B\beta}\}$, even for a flat space case. In this section we begin by local superalgebras in flat space to extract the basis to describe the Hamiltonian and the σ diffeomorphism constraint, which are the Virasoro constraints. From the canonical bracket of these basis we construct a Courant brackets. Then we show the gauge symmetry transformation rules for R-R gauge fields by using this Courant bracket.

2.1 F-string

The local superalgebra for a IIB F-string in flat space is given as

$$\begin{aligned} \{d_{A\alpha}(\sigma), d_{B\beta}(\sigma')\} &= [\delta_{AB} p_{\alpha\beta}(\sigma) + (\tau_3)_{AB} \partial_\sigma x_{\alpha\beta}(\sigma)] \delta(\sigma - \sigma') \\ &= Z_M(\sigma) \Gamma_{AB;\alpha\beta}^M \delta(\sigma - \sigma') , \end{aligned} \quad (2.1)$$

with $p_{\alpha\beta} = p_m \gamma^m{}_{\alpha\beta}$, $x_{\alpha\beta} = x^m \gamma_{m\alpha\beta}$ and $Z_M = (p_m, \partial_\sigma x^m)$. $(\Gamma^M)_{AB;\alpha\beta} = (\delta_{AB} \gamma^m{}_{\alpha\beta}, \tau_{3AB} \gamma_{m\alpha\beta})$ is a gamma matrix for the type II theories satisfying $\{\Gamma^M, \Gamma^N\} = 2\tilde{\delta}^{MN}$. In the right hand side of (2.1) fermionic coordinates are set to be zero, and we will focus only on the bosonic part in this paper. The canonical bracket is given by $\{p_m(\sigma), \partial_\sigma x^n(\sigma')\} = i\delta_m^n \partial_\sigma \delta(\sigma - \sigma')$, where a curly bracket $\{\cdot, \cdot\}$ is used for a distinction from the Courant bracket during this paper.

Hamiltonian is a linear combination of the τ -diffeomorphism \mathcal{H}_\perp and the σ diffeomorphism constraints \mathcal{H}_\parallel . For simplicity we call \mathcal{H}_\perp Hamiltonian from now on, where it is the case in the conformal gauge. The Hamiltonian and the σ -diffeomorphism constraint are given by

$$\left\{ \begin{array}{lcl} \mathcal{H}_\perp & = & \frac{1}{2} \frac{1}{32} \text{tr}(Z_M \Gamma^M)^2 \\ & = & \frac{1}{2} Z_M \tilde{\delta}^{MN} Z_N = \frac{1}{2} (p^2 + (\partial_\sigma x)^2) = 0 , \quad \tilde{\delta}^{MN} = \begin{pmatrix} \eta^{mn} & 0 \\ 0 & \eta_{mn} \end{pmatrix} \\ \mathcal{H}_\parallel & = & \frac{1}{2} Z_M \tilde{\eta}^{MN} Z_N = \partial_\sigma x^m p_m = 0 , \quad \tilde{\eta}^{MN} = \begin{pmatrix} 0 & \delta_m^n \\ \delta_n^m & 0 \end{pmatrix} \end{array} \right. \quad (2.2)$$

where $\tilde{\eta}^{MN}$ is the $O(d,d)$ invariant metric while η^{mn} and η_{mn} are d -dimensional Minkowski metrics.

Let us consider a geometry generated by Z_M which satisfies the following algebra

$$\{Z_M(\sigma), Z_N(\sigma')\} = i\eta_{MN} \partial_\sigma \delta(\sigma - \sigma') , \quad \eta_{MN} = \begin{pmatrix} 0 & \delta_m^n \\ \delta_n^m & 0 \end{pmatrix} . \quad (2.3)$$

The right hand side is the stringy anomalous term, which is proportional to the $O(d,d)$ invariant metric η_{MN} . $\tilde{\eta}^{MN}$ and η_{MN} are introduced independently, however they coincide with each other for the F-string case relating the gauge symmetry and the T-duality consistently. The “operator” Z_M is supposed to act as the derivative with respect to “double field space coordinates” [9] as

$$\{Z_M, f(x, \tilde{x})\}_{double} = -i\partial_M f \equiv (\partial_m f, \partial^m f) = -i \left(\frac{\partial}{\partial x^m} f, \frac{\partial}{\partial \tilde{x}_m} f \right) . \quad (2.4)$$

Local parameters $\Lambda(x)$ ’s are introduced in a vector form $\hat{V} = V^M Z_M$ as

$$\hat{\Lambda}(\sigma) = \Lambda^M Z_M = \Lambda^m p_m + \Lambda_m \partial_\sigma x^m = (\Lambda^m, \Lambda_m) . \quad (2.5)$$

The canonical commutator between two $\hat{\Lambda}$ ’s is calculated as [16]

$$\begin{aligned} \{\hat{\Lambda}_1(\sigma), \hat{\Lambda}_2(\sigma')\} &= -i \left(\Lambda_{[1}^M \partial_M \Lambda_{2]}^N Z_N - \frac{1}{2} \Lambda_{[1}^M \partial_\sigma \Lambda_{2]M} - K \partial_\sigma \Psi_{(12)} \right) \delta(\sigma - \sigma') \\ &\quad + i \left(\left(\frac{1}{2} + K \right) \Psi_{(12)}(\sigma) + \left(\frac{1}{2} - K \right) \Psi_{(12)}(\sigma') \right) \partial_\sigma \delta(\sigma - \sigma') \end{aligned} \quad (2.6)$$

with a symmetric product

$$\Psi_{(12)} = \Lambda_{(1}^m \Lambda_{2)m} = \frac{1}{2} \Lambda_{(1}^M \Lambda_{2)}^N \eta_{NM} = \frac{1}{2} \Lambda_{(1}^M \Lambda_{2)M} . \quad (2.7)$$

The $O(d,d)$ invariant metric η_{MN} is used for lowering indices M . The coefficient K is an arbitrary number corresponding to an ambiguity of the total derivative $\partial_\sigma \Psi_{(12)}$ caused from a term containing $\partial_\sigma \delta(\sigma - \sigma')$. The condition $\partial^m \Lambda = 0$ leads to $\partial_\sigma \Lambda = \partial_\sigma x^m \partial_m \Lambda = Z_M \eta^{MN} \partial_N \Lambda$. Then the commutator (2.6) becomes the Z_M algebra with anomalous term

$$\begin{aligned} \{\hat{\Lambda}_1(\sigma), \hat{\Lambda}_2(\sigma')\} &= -i \hat{\Lambda}_{12}(\sigma) \delta(\sigma - \sigma') \\ &\quad + i \left(\left(\frac{1}{2} + K \right) \Psi_{(12)}(\sigma) + \left(\frac{1}{2} - K \right) \Psi_{(12)}(\sigma') \right) \partial_\sigma \delta(\sigma - \sigma') \\ \hat{\Lambda}_{12} &= \Lambda_{12}^N Z_N \\ \Lambda_{12}^N &= \Lambda_{[1}^M \partial_M \Lambda_{2]}^N - \frac{1}{2} \Lambda_{[1}^M \partial^N \Lambda_{2]M} - K \partial^N \Psi_{(12)} . \end{aligned} \quad (2.8)$$

The regular coefficient of (2.8) is called the C-bracket in a double field space, $\Lambda_{12}^N = ([\Lambda_1, \Lambda_2]_C)^N$. Especially

$$\left([\hat{\Lambda}_1, \hat{\Lambda}_2]_C \right)^N = \begin{cases} \Lambda_{[1}^M \partial_M \Lambda_{2]}^N - \frac{1}{2} \Lambda_{[1}^M \partial^N \Lambda_{2]M} & \cdots \quad K = 0 \\ \Lambda_1^M \partial_M \Lambda_2^N + \Lambda_2^M \partial_{[L} \Lambda_{1|M]} \eta^{LN} & \cdots \quad K = -\frac{1}{2} \end{cases} \quad (2.9)$$

The Jacobiator of the algebra is calculated in terms of the doubled indices as

$$\begin{aligned} & \left[\left[\int \hat{\Lambda}_1, \int \hat{\Lambda}_2 \right]_C, \int \hat{\Lambda}_3 \right]_C + \text{cyclic sum} = - \int \hat{\Lambda}_{[123]} \\ & \hat{\Lambda}_{[123]} = \Lambda_{[123]}^N Z_N = \frac{1}{4} \partial^N \left(\Lambda_{[1}^L \Lambda_{2]}^M \partial_M \Lambda_{3]L} \right) Z_N = \partial_\sigma \left(\frac{1}{4} \Lambda_{[1}^L \Lambda_{2]}^M \partial_M \Lambda_{3]L} \right) \quad , \quad (2.10) \end{aligned}$$

which is independent of the value K . The breakdown of the Jacobi identity is given by a total derivative term so it does not cause serious inconsistency in general.

The gauge symmetry generator is invariant under a further gauge symmetry

$$\begin{aligned} \delta \Lambda^M &= \partial^M \zeta \\ \delta \int \hat{\Lambda} &= \int Z_M \partial^M \zeta = i \int d\sigma \{ \mathcal{H}_\parallel, \zeta \}_{\text{double}} = i \int d\sigma \partial_\sigma \zeta \quad , \quad (2.11) \end{aligned}$$

which vanishes for a closed string. In the second equality of (2.11) we used the fact that \mathcal{H}_\parallel is the σ -diffeomorphism constraint in (2.2) and (2.4). When the parameter satisfies $\partial^m \zeta = 0$, the double field space bracket is reduced to the usual canonical bracket.

The C-bracket is reduced to the Courant bracket under the assumption $\partial^m \lambda = 0$. Let us introduce a tangent vector $\lambda \in T$ and a cotangent vector $\lambda^* \in T^*$

$$\begin{aligned} \hat{\Lambda} &= \lambda + \lambda^* \quad , \quad \lambda = \Lambda^m p_m \quad , \quad \lambda^* = \Lambda_m dx^m \\ \left[\hat{\Lambda}_1, \hat{\Lambda}_2 \right]_{\text{Courant}} &= [\lambda_1, \lambda_2] + \mathcal{L}_{\lambda_1} \lambda_2^* - \mathcal{L}_{\lambda_2} \lambda_1^* - \frac{1}{2} d(\iota_{\lambda_1} \lambda_2^* - \iota_{\lambda_2} \lambda_1^*) \quad (2.12) \\ \begin{cases} [\lambda_1, \lambda_2] &= \Lambda_{[1}^m \partial_m \Lambda_{2]}^n p_n \\ \mathcal{L}_{\lambda_1} \lambda_2^* &= (\Lambda_1^m \partial_m \Lambda_{2;n} + (\partial_n \Lambda_1^m) \Lambda_{2;m}) \partial_\sigma x^n \\ d(\iota_{\lambda_1} \lambda_2^*) &= \partial_n (\Lambda_1^m \Lambda_{2;m}) \partial_\sigma x^n \end{cases} \end{aligned}$$

for $K = 0$. The Courant bracket for $K = -1/2$ is given by;

$$\begin{aligned} \left[\hat{\Lambda}_1, \hat{\Lambda}_2 \right]_{\text{Courant}} &= [\lambda_1, \lambda_2] + \mathcal{L}_{\lambda_1} \lambda_2^* - \iota_{\lambda_2} d\lambda_1^* \quad (2.13) \\ \begin{cases} \iota_{\lambda_2} d\lambda_1^* &= \Lambda_2^m \partial_{[n} \Lambda_{1|m]} \partial_\sigma x^n \end{cases} \end{aligned}$$

The last term is the gauge transformation of the antisymmetric gauge field.

It was shown in [4, 5] that the gauge transformation rule of G_{mn} and B_{mn} are given by the bracket between the “two-vierbein vector” (e_a^m, e_{ma}) and a gauge parameter vector (ξ^m, ξ_m) . Originally Siegel called the bracket a “new Lie derivative” which is recognized as the C-bracket [9]. The two-vierbein fields are transformed linearly under $O(d,d)$ leading to the fractional linearly transformation of $G_{mn} + B_{mn}$ [2, 3] as:

$$G_{mn} + B_{mn} = e_{mn} = e_{ma} e_n^a, \quad e_a^m e_m^b = \delta_a^b, \quad e_m^a e_a^n = \delta_m^n \quad (2.14)$$

$$\hat{e}_a \rightarrow \hat{e}'_a = \begin{pmatrix} e'_a^m \\ e'_{ma} \end{pmatrix} = \begin{pmatrix} A^m_l & B^{ml} \\ C_{ml} & D_m^l \end{pmatrix} \begin{pmatrix} e_a^l \\ e_{la} \end{pmatrix}, \quad \begin{pmatrix} A & B \\ C & D \end{pmatrix} \in O(d,d)$$

$$\begin{pmatrix} e'_a{}^m \\ e'_{ma} \end{pmatrix} e'_n{}^a = \begin{pmatrix} \delta_n^m \\ e'_{mn} \end{pmatrix} \Rightarrow e'_{mn} = (C_{ml} + D_m{}^p e_{pl}) (A^n{}_l + B^{nq} e_{ql})^{-1} . \quad (2.15)$$

The Courant bracket (2.13) between the two-vierbein field and the gauge parameter is given by

$$\begin{aligned} \delta_\xi \hat{e}_a &= [\hat{\xi}, \hat{e}_a]_{Courant} , \quad \hat{e}_a = \begin{pmatrix} e_a{}^m \\ e_{ma} \end{pmatrix} , \quad \hat{\xi} = \begin{pmatrix} \xi^m \\ \xi_m \end{pmatrix} \\ \begin{cases} \delta_\xi e_a{}^m &= \xi^n \partial_n e_a{}^m - e_a{}^n \partial_n \xi^m \\ \delta_\xi e_{ma} &= \xi^n \partial_n e_{ma} + e_{na} \partial_m \xi^n + e_a{}^n \partial_{[m} \xi_{n]} \end{cases} . \end{aligned} \quad (2.16)$$

From the relation (2.14) the transformation (2.16) gives the gauge transformation rules for G_{mn} and B_{mn} as

$$\Rightarrow \begin{cases} \delta_\xi G_{mn} &= \xi^l \partial_l G_{mn} + \partial_{(m} \xi^l G_{l|n)} \\ \delta_\xi B_{mn} &= \xi^l \partial_l B_{mn} + \partial_{[m} \xi^l B_{l|n]} + \partial_{[m} \xi_{n]} \end{cases} . \quad (2.17)$$

This two-vierbein formalism is essential to extend D-brane systems as we will see in the next subsection.

2.2 D3-brane

The local superalgebra for D3-brane in flat space is given by [18]

$$\begin{aligned} &\{d_{A\alpha}(\sigma), d_{B\beta}(\sigma')\} \\ &= [\delta_{AB} p_{\alpha\beta}(\sigma) + (\tau_3)_{AB} q_{\alpha\beta}^{NS}(\sigma) + (\tau_1)_{AB} q_{\alpha\beta}^{R;[1]}(\sigma) + (i\tau_2)_{AB} q_{\alpha\beta}^{R;[3]}(\sigma) + c_{AB\alpha\beta} \Phi(\sigma)] \delta(\sigma - \sigma') \\ &= Z_M(\sigma) \Gamma_{AB;\alpha\beta}^M \delta(\sigma - \sigma') , \\ &p_{\alpha\beta} = p_m (\gamma^m)_{\alpha\beta} , \quad q_{\alpha\beta}^{R;[1]} = \epsilon^{ijk} F_{ij} \partial_k x^m (\gamma_m)_{\alpha\beta} \\ &q_{\alpha\beta}^{NS} = E^i \partial_i x^m (\gamma_m)_{\alpha\beta} , \quad q_{\alpha\beta}^{R;[3]} = \epsilon^{ijk} \partial_i x^m \partial_j x^n \partial_k x^l (\gamma_{mnl})_{\alpha\beta} , \end{aligned} \quad (2.18)$$

with the Gauss law constraint

$$\Phi = \partial_i E^i = 0 . \quad (2.19)$$

E^i and $F_{ij} = \partial_{[i} A_{j]}$ are Dirac-Born-Infeld (DBI) U(1) electric field and magnetic field on a D3-brane and $c_{AB\alpha\beta}$ is a function. We focus on the bosonic part only. Let us consider the algebra generated by

$$Z_M = \begin{pmatrix} p_m \\ E^i \partial_i x^m \\ \epsilon^{ijk} F_{ij} \partial_k x^m \\ \epsilon^{ijk} \partial_i x^m \partial_j x^n \partial_k x^l \end{pmatrix} \quad (2.20)$$

which is a vector of $T \oplus T^* \oplus \Lambda^1 T^* \oplus \Lambda^3 T^*$ [12]. The upper half is the NS-NS sector and lower half is the R-R sector. The normalization in (2.20) is omitted, while the correct coefficient is given by (A.2) and (A.3) in the appendix. The Hamiltonian is the sum of bilinears in Z_M

$$\left\{ \begin{array}{lcl} \mathcal{H}_\perp & = & \frac{1}{2} \frac{1}{32} \text{tr}(Z_M \Gamma^M)^2 = \frac{1}{2} Z_M \tilde{\delta}^{MN} Z_N \\ \\ & = & \frac{1}{2} \left(p^2 + (E^i \partial_i x)^2 + (\epsilon^{ijk} F_{ij} \partial_k x^m)^2 + (\epsilon^{ijk} \partial_i x^m \partial_j x^n \partial_k x^l)^2 \right) = 0 \\ \\ \mathcal{H}_i & = & \partial_i x^m p_m + F_{ij} E^j = 0 \end{array} \right. .$$

The worldvolume diffeomorphism constraints $\mathcal{H}_i = 0$ can be written in a bilinear form of Z_M by contracting with E^i , $\epsilon^{ijk} F_{jk}$ and $\epsilon^{ijk} \partial_j x^m \partial_k x^n$ as

$$E^i \mathcal{H}_i = \epsilon^{ijk} F_{ij} \mathcal{H}_k = \epsilon^{ijk} \partial_i x^n \partial_j x^l \mathcal{H}_k = 0$$

$$\Rightarrow Z_M \tilde{\rho}^{MN} Z_N = 0 , \quad \tilde{\rho}^{MN} = \left(\begin{array}{cc|cc} 0 & a \delta_m^n & b \delta_n^m & c_{[n_1 n_2} \delta_{n]}^m \\ a \delta_m^n & 0 & c_{mn} & 0 \\ \hline b \delta_m^n & c_{mn} & 0 & \\ c_{[m_1 m_2} \delta_{m]}^n & 0 & & \end{array} \right) \quad (2.21)$$

with arbitrary coefficients a, b, c_{mn} .

The algebra generated by Z_M is closed by the Gauss law constraint. The D3-brane extension of (2.3) is given by

$$\{Z_M(\sigma), Z_N(\sigma')\} = i \rho_{MN}^i \partial_i \delta^{(3)}(\sigma - \sigma')$$

$$\rho_{MN}^i = \left(\begin{array}{cc|cc} 0 & E^i \delta_m^n & \epsilon^{ijk} F_{jk} \delta_m^n & \frac{1}{2} \epsilon^{ijk} \partial_j x^{[n_1} \partial_k x^{n_2]} \delta_m^n \\ E^i \delta_n^m & 0 & \epsilon^{ijk} \partial_j x^{[m} \partial_k x^{n]} & 0 \\ \hline \partial^{ijk} F_{jk} \delta_n^m & \epsilon^{ijk} \partial_j x^{[n} \partial_k x^{m]} & & \\ \frac{1}{2} \epsilon^{ijk} \partial_j x^{[m_1} \partial_k x^{m_2]} \delta_n^m & 0 & & \end{array} \right) \quad (2.22)$$

with $\mathcal{O}_{[abc]} = \mathcal{O}_{a[bc]} + \mathcal{O}_{b[ca]} + \mathcal{O}_{c[ab]}$. The Gauss law and the Bianchi identity guarantee that the right hand side of (2.22) is a total derivative. Unlike the F-string case, for a D-brane case $\tilde{\rho}^{MN}$ in (2.21) relating to T-duality and ρ_{MN}^i in (2.22) relating to gauge symmetry neither coincide nor manifest the $O(d,d)$ symmetry.

Let us write down a canonical commutator between two vectors

$$\hat{\Lambda}_I(\sigma) = \Lambda_I^M Z_M = (\Lambda_I^n, \Lambda_{In}, \tilde{\Lambda}_{In}, \Lambda_{Inrs}) \quad (2.23)$$

as

$$\begin{aligned}
\{\hat{\Lambda}_1(\sigma), \hat{\Lambda}_2(\sigma')\} &= -i\hat{\Lambda}_{12}(\sigma)\delta^{(3)}(\sigma - \sigma') \\
&\quad + i\left(\left(\frac{1}{2} + K\right)\Psi_{(12)}^i(\sigma) + \left(\frac{1}{2} - K\right)\Psi_{(12)}^i(\sigma')\right)\partial_i\delta^{(3)}(\sigma - \sigma') \quad (2.24)
\end{aligned}$$

$$\left\{
\begin{aligned}
\Lambda_{12}^n &= \Lambda_{[1}^m\partial_m\Lambda_{2]}^n \\
\Lambda_{12;n} &= \Lambda_{[1}^m\partial_m\Lambda_{2]n} - \frac{1}{2}(\Lambda_{[1}^m\partial_n\Lambda_{2]m} + \Lambda_{[1|m}\partial_n\Lambda_{2]m}^m) - K\partial_n(\Lambda_{(1}^m\Lambda_{2)m}) \\
\tilde{\Lambda}_{12;n} &= \Lambda_{[1}^m\partial_m\tilde{\Lambda}_{2]n} - \frac{1}{2}(\Lambda_{[1}^m\partial_n\tilde{\Lambda}_{2]m} + \tilde{\Lambda}_{[1|m}\partial_n\Lambda_{2]m}^m) - K\partial_n(\Lambda_{(1}^m\tilde{\Lambda}_{2)m}) \\
\Lambda_{12;nlr} &= \Lambda_{[1}^m\partial_m\Lambda_{2]nlr} - \frac{1}{4}\left(\Lambda_{[1}^m\partial_{[n}\Lambda_{2]m]lr} - (\partial_{[n}\Lambda_{[1}^m)\Lambda_{2]m]lr}\right) - \frac{K}{2}\partial_{[n}(\Lambda_{(1}^m\Lambda_{2)m]lr}) \\
&\quad + \frac{1}{6}\left(\Lambda_{[n}^1\partial_l\tilde{\Lambda}_{r]}^2 + \partial_{[n}\Lambda_l^1\tilde{\Lambda}_{r]}^2\right) - \frac{K}{3}\partial_{[n}(\Lambda_l^1\tilde{\Lambda}_{r]}^2) \\
\Psi_{(12)}^i &= \frac{1}{2}\Lambda_{(1}^M\Lambda_{2)}^N\rho_{MN}^i
\end{aligned}
\right. \quad .$$

where $\Psi_{(12)}^i$ is an ambiguity caused from $\partial_i\delta^{(3)}(\sigma - \sigma')$.

Now we refer to the coefficient $\hat{\Lambda}_{12}(\sigma)$ in (2.24) as the Courant bracket for D3-brane analogously to the previous section. A vector in Z_M space is denoted by

$$\hat{\Lambda} = \lambda + \lambda^* + \lambda^{[1]} + \lambda^{[3]} \in T \oplus T^* \oplus \Lambda^1 T^* \oplus \Lambda^3 T^*$$

then $\hat{\Lambda}_{12}(\sigma)$ in (2.24) is recognized as the Courant bracket for D3-brane with $K = 0$ as ;

$$\begin{aligned}
[\hat{\Lambda}_1, \hat{\Lambda}_2]_{D3} &= [\lambda_1, \lambda_2] + \mathcal{L}_{\lambda_{[1}}\lambda_{2]}^* + \mathcal{L}_{\lambda_{[1}}\lambda_{2]}^{[1]} + \mathcal{L}_{\lambda_{[1}}\lambda_{2]}^{[3]} - \frac{1}{2}d\left(\iota_{\lambda_{[1}}\lambda_{2]}^* + \iota_{\lambda_{[1}}\lambda_{2]}^{[1]} + \iota_{\lambda_{[1}}\lambda_{2]}^{[3]}\right) \\
&\quad + \frac{1}{6}\left(\lambda_{[1}^*\wedge d\lambda_{2]}^{[1]} + d\lambda_{[1}^*\wedge \lambda_{2]}^{[1]}\right) \quad (2.25)
\end{aligned}$$

$$\left\{
\begin{aligned}
[\lambda_1, \lambda_2] &= \Lambda_{[1}^m\partial_m\Lambda_{2]}^n p_n \\
\mathcal{L}_{\lambda_1}\lambda_2^* &= (\Lambda_1^m\partial_m\Lambda_{2;n} + (\partial_n\Lambda_1^m)\Lambda_{2;m}) E^i\partial_i x^n \\
\mathcal{L}_{\lambda_1}\lambda_2^{[1]} &= \left(\Lambda_1^m\partial_m\tilde{\Lambda}_{2;n} + (\partial_n\Lambda_1^m)\tilde{\Lambda}_{2;m}\right) \epsilon^{ijk}F_{ij}\partial_k x^n \\
\mathcal{L}_{\lambda_1}\lambda_2^{[3]} &= \left(\Lambda_1^m\partial_m\Lambda_{2;nlr} + \frac{1}{2}(\partial_{[n}\Lambda_1^m)\Lambda_{2;m]lr}\right) \epsilon^{ijk}\partial_i x^n\partial_j x^l\partial_k x^r \\
d(\iota_{\lambda_1}\lambda_2^*) &= \partial_n(\Lambda_1^m\Lambda_{2;m}) E^i\partial_i x^n \\
d(\iota_{\lambda_1}\lambda_2^{[1]}) &= \partial_n(\Lambda_1^m\tilde{\Lambda}_{2;m}) \epsilon^{ijk}F_{ij}\partial_k x^n \\
d(\iota_{\lambda_1}\lambda_2^{[3]}) &= \partial_n(\Lambda_1^m\Lambda_{2;m}lr) \epsilon^{ijk}\partial_i x^n\partial_j x^l\partial_k x^r \\
\lambda_1^*\wedge d\lambda_2^{[1]} &= \Lambda_{[n}^1\partial_l\tilde{\Lambda}_{r]}^2\epsilon^{ijk}\partial_i x^n\partial_j x^l\partial_k x^r \\
d\lambda_1^*\wedge \lambda_2^{[1]} &= (\partial_{[n}\Lambda_l^1)\tilde{\Lambda}_{r]}^2\epsilon^{ijk}\partial_i x^n\partial_j x^l\partial_k x^r
\end{aligned}
\right. \quad (2.26)$$

It is also convenient to introduce the Courant bracket for D3-brane with $K = -1/2$;

$$\begin{aligned}
[\hat{\Lambda}_1, \hat{\Lambda}_2]_{D3} &= [\lambda_1, \lambda_2] + \mathcal{L}_{\lambda_1}\lambda_2^* + \mathcal{L}_{\lambda_1}\lambda_2^{[1]} + \mathcal{L}_{\lambda_1}\lambda_2^{[3]} - \iota_{\lambda_2}d\lambda_1^* - \iota_{\lambda_2}d\lambda_1^{[1]} - \iota_{\lambda_2}d\lambda_1^{[3]} \\
&\quad + \frac{1}{3}\left(\lambda_2^{[1]}\wedge d\lambda_1^* - \lambda_2^*\wedge d\lambda_1^{[1]}\right) \quad (2.27)
\end{aligned}$$

$$\begin{cases} \iota_{\lambda_2} d\lambda_1^* = \Lambda_2^m \partial_{[m} \Lambda_{1;n]} E^i \partial_i x^n \\ \iota_{\lambda_2} d\lambda_1^{[1]} = \Lambda_2^m \partial_{[m} \tilde{\Lambda}_{1;n]} \epsilon^{ijk} F_{ij} \partial_k x^n \\ \iota_{\lambda_2} d\lambda_1^{[3]} = \frac{1}{3!} \Lambda_2^m \partial_{[m} \Lambda_{1;nlr]} \epsilon^{ijk} \partial_i x^n \partial_j x^l \partial_k x^r \end{cases} \quad (2.28)$$

In our canonical approach the appearance of the Chern-Simons terms, as shown to exist in [14], in the second lines of (2.25) and (2.27) comes from the canonical commutator between the DBI U(1) fields.

The Jacobi identity is also broken by a total derivative term, since the D3 extended space vector $\hat{\Lambda}$ can be also written as

$$\hat{\Lambda} = \Lambda^m p_m + \Lambda_m^i \partial_i x^m, \quad \Lambda_m^i = \Lambda_m E^i + \tilde{\Lambda}_m \epsilon^{ijk} F_{jk} + \Lambda_{mnl} \epsilon^{ijk} \partial_j x^n \partial_k x^l \quad (2.29)$$

which leads to a total derivative term as the Jacobiator as seen in (2.10).

Next we examine the gauge transformation rules for the R-R gauge fields. We extend the NS-NS gauge fields to the R-R gauge fields for D3-brane as

$$\hat{e}_a = \begin{pmatrix} e_a^m \\ e_{ma} \end{pmatrix} \Rightarrow \hat{C}_a^{D3} = \begin{pmatrix} e_a^m \\ \frac{e_{ma} = e_a^l (B_{ml} + G_{ml})}{e_a^l C_{ml}^{[2]}} \\ e_a^p C_{mnl}^{[4]} \end{pmatrix}. \quad (2.30)$$

Then the gauge transformations for the R-R gauge fields are given by the Courant bracket in (2.28) with the parameter as

$$\begin{aligned} \delta_\xi \hat{C}_a^{D3} &= [\hat{\xi}, \hat{C}_a^{D3}]_{D3}, \quad \hat{\xi} = \begin{pmatrix} \xi^m \\ \xi_m \\ \xi_m^{[1]} \\ \xi_{mnl}^{[3]} \end{pmatrix} \\ \Rightarrow \quad \begin{cases} \delta_\xi C^{[2]} = \mathcal{L}_\xi C^{[2]} + d\xi^{[1]} \\ \delta_\xi C^{[4]} = \mathcal{L}_\xi C^{[4]} - C^{[2]} \wedge d\xi + d\xi^{[3]} + B \wedge d\xi^{[1]} \end{cases} \end{aligned} \quad (2.31)$$

where $\xi = \xi_m$ is the gauge parameter for B field. The gauge transformation of $C^{[4]}$ involves the B field as expected.

3 Strings in curved background

In the previous section we have shown three points: the canonical approach to the Hamiltonian; the local superalgebra gives a Courant bracket for D3-brane; and the gauge symmetry

of the R-R gauge fields are obtained by our Courant bracket. The two-vierbein formalism of the NS-NS gauge fields, G_{mn} and B_{mn} , constructs the O(d,d) vector manifesting the T-duality transformation and the gauge transformation. A similar mechanism seems to work for the R-R gauge fields. In order to analyze the R-R couplings directly we begin by strings in a curved background clarifying the background fields dependence.

3.1 F-string

The bosonic part of the action for a F-string in a curved space is given by

$$\begin{aligned} I &= \int d^2\sigma \mathcal{L} , \quad \mathcal{L} = \mathcal{L}_{NG} + \mathcal{L}_{WZ} \\ \mathcal{L}_{NG} &= -T_{F1}\sqrt{-h} , \quad h = \det h_{\mu\nu} , \quad h_{\mu\nu} = \partial_\mu x^m \partial_\nu x^n G_{mn} \\ \mathcal{L}_{WZ} &= \frac{1}{2}T_{F1}\epsilon^{\mu\nu}\partial_\mu x^m \partial_\nu x^n B_{mn} . \end{aligned}$$

where $T_{F1} = \frac{1}{2\pi\alpha'}$ and B_{mn} is the NS-NS two-form gauge field. The canonical momentum is defined as

$$p_m \equiv \frac{\partial \mathcal{L}}{\partial(\partial_0 x^m)} = -T_{F1}\sqrt{-h}h^{0\mu}\partial_\mu x^n G_{mn} + T_{F1}\epsilon^{01}\partial_\sigma x^n B_{mn} , \quad (3.1)$$

where $h^{\mu\nu}$ is the inverse of $h_{\mu\nu}$.

The Hamiltonian is obtained by the Legendre transformation as

$$\begin{aligned} H &= \int d\sigma \mathcal{H} \\ \mathcal{H} &= p_m \partial_0 x^m - \mathcal{L} = -\frac{1}{\sqrt{-h}h^{00}}\mathcal{H}_\perp - \frac{h^{01}}{h^{00}}\mathcal{H}_\parallel \\ \begin{cases} \mathcal{H}_\perp &= \frac{1}{2T_{F1}}(\tilde{p}_m G^{mn} \tilde{p}_n + T_{F1}^2 h_{11}) = 0 \\ \mathcal{H}_\parallel &= \partial_\sigma x^m \tilde{p}_m = \partial_\sigma x^m p_m = 0 \end{cases} \end{aligned}$$

with

$$\tilde{p}_m \equiv p_m - T_{F1}\epsilon^{01}\partial_\sigma x^n B_{mn} . \quad (3.2)$$

By virtue of the bilinear expression $h_{11} = x'^m G_{mn} x'^n$, \mathcal{H}_\perp is recast into the sum of bilinears as

$$\begin{aligned} \mathcal{H}_\perp &= \frac{1}{2T_{F1}}(\tilde{p}_m T_{F1}\partial_\sigma x^m) \begin{pmatrix} G^{mn} & 0 \\ 0 & G_{mn} \end{pmatrix} \begin{pmatrix} \tilde{p}_n \\ T_{F1}\partial_\sigma x^n \end{pmatrix} \\ &= \frac{1}{2T_{F1}} Z_M^T \mathcal{M}^{MN} Z_N \end{aligned}$$

$$\left\{ \begin{array}{l} Z_N = \begin{pmatrix} p_n \\ T_{F1} \partial_\sigma x^n \end{pmatrix} \\ \mathcal{M}^{MN} = \begin{pmatrix} \delta^m{}_p & 0 \\ B_{mp} & \delta_m{}^p \end{pmatrix} \begin{pmatrix} G^{pq} & 0 \\ 0 & G_{pq} \end{pmatrix} \begin{pmatrix} \delta_q{}^n & -B_{qn} \\ 0 & \delta^q{}_n \end{pmatrix} \\ = \begin{pmatrix} G^{mn} & -G^{mq}B_{qn} \\ B_{mp}G^{pn} & G_{mn} - B_{mp}G^{pq}B_{qn} \end{pmatrix} \end{array} \right. . \quad (3.3)$$

The $Z_M = Z_M(\sigma)$ base contains only the worldvolume variables, while $\mathcal{M}^{MN} = \mathcal{M}^{MN}(x)$ contains only the spacetime background fields. It is further written as

$$\begin{aligned} \mathcal{M}^{MN} &= \tilde{\delta}^{AB} E_A{}^M E_B{}^N \\ G^{pq} &= \eta^{ab} e_a{}^p e_b{}^q \quad , \quad E_A{}^M = \begin{pmatrix} e_a{}^q & 0 \\ 0 & e^a{}_q \end{pmatrix} \begin{pmatrix} \delta_q{}^n & -B_{qn} \\ 0 & \delta^q{}_n \end{pmatrix} \quad , \\ G_{pq} &= \eta_{ab} e_a{}^p e_b{}^q \end{aligned} \quad (3.4)$$

and

$$\mathcal{H}_\parallel = \frac{1}{2} Z_M \tilde{\eta}^{MN} Z_N \quad , \quad \tilde{\eta}^{MN} = \begin{pmatrix} 0 & \delta_n^m \\ \delta_m^n & 0 \end{pmatrix} \quad . \quad (3.5)$$

There exists the $O(d,d)$ symmetry which preserves $\tilde{\eta}^{MN}$ for a string background. The T-duality transformation of the background fields for a bosonic string is the $O(d,d)$ transformation. The subgroup which preserve both $\tilde{\eta}^{MN}$ and $\tilde{\delta}^{MN}$ is $O(d) \times O(d)$. $E_A{}^M$ is a coset element of $O(d,d)/O(d) \times O(d)$ whose dimension is the same as $GL(d)$ as well as the one of the two-vierbein field \hat{e}_a .

3.2 AdS-string

It is possible to extend the subsection 2.1 to a string on AdS (or sphere) space which is described by a group manifold G . For a group element $X \in G$ the left invariant one-form and the covariant derivative are given as

$$J^a = X^{-1} dX = dx^m e_m{}^a \quad , \quad D_a = e_a{}^m p_m \quad , \quad e_m{}^a e_a{}^n = \delta_m^n \quad , \quad e_a{}^m e_m{}^b = \delta_a^b \quad . \quad (3.6)$$

The Hamiltonian and the σ -diffeomorphism constraint are written in terms of $Z_A = (D_a, J^a)$ as

$$\left\{ \begin{array}{l} \mathcal{H}_\perp = \frac{1}{2} Z_A \tilde{\delta}^{AB} Z_B = \frac{1}{2} \left((D_a)^2 + (J^a)^2 \right) = 0 \\ \mathcal{H}_\parallel = \frac{1}{2} Z_A \tilde{\eta}^{AB} Z_B = J^a D_a = \partial_\sigma x^m p_m = 0 \end{array} \right.$$

with $\tilde{\delta}^{AB} = \text{diag}(\eta^{ab}, \eta_{ab})$.

Let us consider a space generated by Z_A satisfying the following algebra [26]

$$\begin{aligned} \{Z_A(\sigma), Z_B(\sigma')\} &= -iF_{AB}^C Z_C \delta(\sigma - \sigma') + i\eta_{AB} \partial_\sigma \delta(\sigma - \sigma') \quad (3.7) \\ F_{AB}^C Z_C &= \begin{pmatrix} f_{ab}^c D_c & -f_{ac}^b J^c \\ f_{bc}^a J^c & 0 \end{pmatrix}, \quad \eta_{AB} = \begin{pmatrix} 0 & \delta_a^b \\ \delta_b^a & 0 \end{pmatrix} \\ f_{ab}^c &= e_{[a}^m \partial_m e_{b]}^n e_n^c \end{aligned}$$

where f_{ab}^c is the structure constant for the group G . A vector $\hat{\Lambda}$ in the space $T \oplus T^*$ is introduced

$$\hat{\Lambda}(\sigma) = \Lambda^A Z_A = \Lambda^a D_a + \Lambda_a J^a = \Lambda^a e_a^m p_m + \Lambda_a e_m^a \partial_\sigma x^m, \quad (3.8)$$

then the canonical bracket between two vectors is given by

$$\begin{aligned} \{\hat{\Lambda}_1(\sigma), \hat{\Lambda}_2(\sigma')\} &= -i\hat{\Lambda}_{12} \delta(\sigma - \sigma') + i \left(\left(\frac{1}{2} + K \right) \Psi_{(12)}(\sigma) + \left(\frac{1}{2} - K \right) \Psi_{(12)}(\sigma') \right) \partial_\sigma \delta(\sigma - \sigma') \\ \Lambda_{12}^A &= \frac{1}{2} \Lambda_{[1}^C \Lambda_{2]}^B F_{CB}^A + \Lambda_{[1}^B \partial_B \Lambda_{2]}^A - \frac{1}{2} \Lambda_{[1}^B \partial^A \Lambda_{2]B} - K \partial^A \Psi_{12} \\ \Psi_{(12)} &= \Lambda_{(1}^a \Lambda_{2)a} = \frac{1}{2} \Lambda_{(1}^M \Lambda_{2)M} \end{aligned}$$

with $\{Z_A, \Lambda\} = -i\partial_A \Lambda = -i(e_a^m \partial_m \Lambda, e_m^a \partial^m \Lambda)$. Now the C-bracket in the double field space is $([\hat{\Lambda}_1, \hat{\Lambda}_2]_C)^A = \Lambda_{12}^A$ which contains the structure constant. A similar bracket is introduced in [6, 21]. If we denote $(\Lambda^a D_a, \Lambda_a J^a) = (\Lambda^a e_a^m p_m, \Lambda_a e_m^a \partial_\sigma x^m) = (\lambda, \lambda^*)$ and impose $\partial^m \Lambda = 0 = \partial^a \Lambda$, then the C-bracket is reduced to the Courant bracket given by (2.12) as

$$\begin{aligned} [\hat{\Lambda}_1, \hat{\Lambda}_2]_{AdS} &= [\lambda_1, \lambda_2] + \mathcal{L}_{\lambda_{[1}} \lambda_{2]}^* - \frac{1}{2} d(\iota_{\lambda_{[1}} \lambda_{2]}^*) \\ \begin{cases} [\lambda_1, \lambda_2] &= \left(\Lambda_{[1}^a \partial_a \Lambda_{2]}^b + \frac{1}{2} \Lambda_{[1}^a \Lambda_{2]}^c f_{ac}^b \right) D_b \\ \mathcal{L}_{\lambda_{[1}} \lambda_{2]}^* &= \left(\Lambda_{[1}^a \partial_a \Lambda_{2]b} - \Lambda_{[1}^a \Lambda_{2]c} f_{ab}^c + \partial_b \Lambda_{[1}^a \Lambda_{2]a} \right) J^b \\ d(\iota_{\lambda_1} \lambda_2^*) &= \partial_a (\Lambda_1^b \Lambda_{2;b}) J^a \end{cases} \end{aligned} \quad (3.9)$$

3.3 D-string

The bosonic part of the action for a D-string in a curved space is given by ¹

$$\begin{aligned} I &= \int d^2\sigma \mathcal{L}, \quad \mathcal{L} = \mathcal{L}_{DBI} + \mathcal{L}_{WZ} \quad (3.10) \\ \mathcal{L}_{DBI} &= -T_{D1} e^{-\phi} \sqrt{-h_F}, \quad h_F = \det h_{F\mu\nu} \end{aligned}$$

¹There is an alternative formulation for a D-brane [27].

$$\begin{aligned}
\mathcal{L}_{WZ} &= \frac{T_{D1}}{2} \epsilon^{\mu\nu} \left(\partial_\mu x^m \partial_\nu x^n C_{mn}^{[2]} + 2\pi\alpha' \mathcal{F}_{\mu\nu} C^{[0]} \right) \\
h_{F\mu\nu} &= \partial_\mu x^m \partial_\nu x^n (G_{mn} + B_{mn}) + 2\pi\alpha' F_{\mu\nu} \\
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu , \quad \mathcal{F}_{\mu\nu} = F_{\mu\nu} + \frac{1}{2\pi\alpha'} \partial_\mu x^m \partial_\nu x^n B_{mn} .
\end{aligned}$$

The canonical momenta are defined as

$$\begin{aligned}
p_m &\equiv \frac{\partial \mathcal{L}}{\partial(\partial_0 x^m)} \\
&= -T_{D1} e^{-\phi} \sqrt{-h_F} \left(\frac{1}{2} h_F^{(\mu 0)} G_{mn} + \frac{1}{2} h_F^{[\mu 0]} B_{mn} \right) \partial_\mu x^n + T_{D1} \epsilon^{01} \partial_\sigma x^n \left(C_{mn}^{[2]} + C^{[0]} B_{mn} \right) \\
E^1 &\equiv \frac{\partial \mathcal{L}}{\partial(\partial_0 A_1)} \\
&= 2\pi\alpha' T_{D1} \left(-e^{-\phi} \sqrt{-h_F} \frac{1}{2} h_F^{[10]} + \epsilon^{01} C^{[0]} \right) .
\end{aligned} \tag{3.11}$$

A matrix $h_F^{\mu\nu}$ is the inverse of $h_{F\mu\nu}$ as $h_{F\mu\nu} h_F^{\nu\rho} = \delta_\mu^\rho$. Symmetrized/antisymmetrized indices are denoted as $h_F^{(\mu 0)} = h_F^{\mu 0} + h_F^{0\mu}$ and $h_F^{[\mu 0]} = h_F^{\mu 0} - h_F^{0\mu}$. The Legendre transformation brings the Lagrangian to the Hamiltonian as

$$\begin{aligned}
H &= \int d\sigma \mathcal{H} \\
\mathcal{H} &= p_m \partial_0 x^m + E^1 \partial_0 A_1 - \mathcal{L} \\
&= -\frac{1}{\sqrt{-h} h^{00}} \mathcal{H}_\perp - \frac{h^{01}}{h^{00}} \mathcal{H}_\parallel - A_0 \Phi \\
&\quad \left\{ \begin{array}{l} \mathcal{H}_\perp = \frac{1}{2T_{D1}} e^\phi \left(\tilde{p}_m G^{mn} \tilde{p}_n + \frac{1}{(2\pi\alpha')^2} \tilde{E}^1 h_{11} \tilde{E}^1 + T_{D1}^2 e^{-2\phi} h_{11} \right) = 0 \\ \mathcal{H}_\parallel = \tilde{p}_m \partial_\sigma x^m = p_m \partial_\sigma x^m = 0 \\ \Phi = \partial_\sigma E^1 = 0 \end{array} \right.
\end{aligned}$$

with

$$\begin{aligned}
\tilde{p}_m &\equiv p_m - B_{mn} \frac{1}{2\pi\alpha'} E^1 \partial_\sigma x^n - C_{mn}^{[2]} T_{D1} \epsilon^{01} \partial_\sigma x^n \\
&= -T_{D1} e^{-\phi} \sqrt{-h_F} \frac{1}{2} h_F^{(\mu 0)} G_{mn} \partial_\mu x^n \\
\tilde{E}^1 &\equiv E^1 - C^{[0]} 2\pi\alpha' T_{D1} \epsilon^{01} = -\frac{2\pi\alpha'}{2} T_{D1} e^{-\phi} \sqrt{-h_F} h_F^{[10]} .
\end{aligned} \tag{3.12}$$

Analogously to the previous section \mathcal{H}_\perp is recast into the sum of bilinears as

$$\mathcal{H}_\perp = \frac{1}{2T_{D1}} Z_M^T \mathcal{M}^{MN} Z_N$$

with

$$Z_N = \begin{pmatrix} p_n \\ \frac{1}{2\pi\alpha'} E^1 \partial_\sigma x^n \\ T_{D1} \partial_\sigma x^n \end{pmatrix} \tag{3.13}$$

$$\begin{aligned}
& \mathcal{M}^{MN} \\
&= \left(\begin{array}{cc|c} \delta_m^m & 0 & \\ B_{mp} & \delta_m^p & \\ \hline \tilde{C}_{mp}^{[2]} & -C^{[0]}\delta_m^p & \delta_m^p \end{array} \right) \left(\begin{array}{cc|c} e^\phi G^{pq} & 0 & \\ 0 & e^\phi G_{pq} & \\ \hline & e^{-\phi} G_{pq} & \end{array} \right) \left(\begin{array}{cc|c} \delta_q^n & -B_{qn} & -\tilde{C}_{qn}^{[2]} \\ 0 & \delta_n^q & -C^{[0]}\delta_n^q \\ \hline & \delta_n^q & \end{array} \right) \\
&= \left(\begin{array}{cc|c} e^\phi G^{mn} & -e^\phi G^{mq}B_{qn} & -e^\phi G^{mq}\tilde{C}_{qn}^{[2]} \\ e^\phi B_{mp}G^{pn} & e^\phi G_{mn} - e^\phi B_{mp}G^{pq}B_{qn} & -e^\phi C^{[0]}G_{mn} - e^\phi B_{mp}G^{pq}\tilde{C}_{qn}^{[2]} \\ e^\phi \tilde{C}_{mp}^{[2]}G^{pn} & -e^\phi C^{[0]}G_{mn} - e^\phi \tilde{C}_{mp}^{[2]}G^{pq}B_{qn} & (e^{-\phi} + e^\phi(C^{[0]})^2)G_{mn} - e^\phi \tilde{C}_{mp}^{[2]}G^{pq}\tilde{C}_{qn}^{[2]} \end{array} \right) \tag{3.14}
\end{aligned}$$

with $\tilde{C}_{mn}^{[2]} = C_{mn}^{[2]} + C^{[0]}B_{mn}$. The R-R coupling is separated as the above. It has the inverse dilaton dependence $e^{-\phi}$. The upper-left part of the \mathcal{M} matrix is the same as the F-string case in (3.3).

The Z_M algebra is given as

$$\{Z_M(\sigma), Z_N(\sigma')\} = i\rho_{MN}\partial_\sigma\delta(\sigma - \sigma') , \quad \rho_{MN} = \left(\begin{array}{cc|c} 0 & \frac{1}{2\pi\alpha'}E^1\delta_m^n & T_{D1}\delta_m^n \\ \frac{1}{2\pi\alpha'}E^1\delta_n^m & 0 & 0 \\ \hline T_{D1}\delta_n^m & 0 & 0 \end{array} \right) . \tag{3.15}$$

The canonical bracket between two vectors $\hat{\Lambda}_I = (\Lambda_I^m, \Lambda_{I;m}, \tilde{\Lambda}_{I;m}) \in T \oplus T^* \oplus \Lambda^1 T^*$ is

$$\begin{aligned}
\{\hat{\Lambda}_1(\sigma), \hat{\Lambda}_2(\sigma')\} &= -i\hat{\Lambda}_{12}\delta(\sigma - \sigma') + i\left((\frac{1}{2} + K)\Psi_{(12)}(\sigma) + (\frac{1}{2} - K)\Psi_{(12)}(\sigma')\right)\partial_\sigma\delta(\sigma - \sigma') \\
\hat{\Lambda}_{12} &= \Lambda_{[1}^m\partial_m\Lambda_{2]}^n p_n \\
&\quad + \left(\Lambda_{[1}^m\partial_m\tilde{\Lambda}_{2]n} - \frac{1}{2}(\Lambda_{[1}^m\partial_n\tilde{\Lambda}_{2]m} - \partial_n\Lambda_{[1}^m\tilde{\Lambda}_{2]m}) - K\partial_n\Psi_{(12)}\right)\partial_\sigma x^n \\
\tilde{\Lambda}_{I;m} &= \frac{E^1}{2\pi\alpha'}\Lambda_{I;m} + T_{D1}\tilde{\Lambda}_{I;m} \\
\Psi_{(12)} &= \Lambda_{(1}^m\tilde{\Lambda}_{2)m} .
\end{aligned} \tag{3.16}$$

Introducing the notation of vectors as

$$\hat{\Lambda}_I = \lambda_I + \lambda_I^* + \lambda_I^{[1]} \in T \oplus T^* \oplus \Lambda^1 T^* , \tag{3.17}$$

we refer to the coefficient $\hat{\Lambda}_{12}$ as the Courant bracket for D-string

$$[\hat{\Lambda}_1, \hat{\Lambda}_2]_{D1} = \begin{cases} [\lambda_1, \lambda_2] + \mathcal{L}_{\lambda_{[1}}\lambda_{2]}^* + \mathcal{L}_{\lambda_{[1}}\lambda_{2]}^{[1]} - \frac{1}{2}d(\iota_{\lambda_{[1}}\lambda_{2]}^* + \iota_{\lambda_{[1}}\lambda_{2]}^{[1]}) & \dots \quad K = 0 \\ [\lambda_1, \lambda_2] + \mathcal{L}_{\lambda_1}\lambda_2^* + \mathcal{L}_{\lambda_1}\lambda_2^{[1]} - \iota_{\lambda_2}d\lambda_1^* - \iota_{\lambda_1}d\lambda_1^{[1]} & \dots \quad K = -\frac{1}{2} \end{cases} \tag{3.18}$$

The Jacobiator follows from (2.10) by replacing $\hat{\Lambda}_I = (\Lambda_I^m, \tilde{\Lambda}_{I;m})$.

The gauge transformation of the R-R gauge field is given by the Courant bracket between a gauge field vector \hat{C}^{D1} and a parameter vector $\hat{\xi}$

$$\hat{C}_a^{D1} = \begin{pmatrix} e_a^m \\ \frac{e_{ma}}{e_a^n C_{mn}^{[2]}} \end{pmatrix}, \quad \hat{\xi} = \begin{pmatrix} \xi^m \\ \frac{\xi_m}{\xi_m^{[1]}} \end{pmatrix}$$

$$\delta_\xi \hat{C}_a^{D1} = [\hat{\xi}, \hat{C}_a^{D1}]_{D1} \Rightarrow \delta_\xi C^{[2]} = \mathcal{L}_\xi C^{[2]} + d\xi^{[1]} \quad (3.19)$$

as well as the one for the NS-NS gauge fields in (2.17) .

4 Dp-branes in curved background

In this section we analyze the Courant bracket for arbitrary type II Dp-branes including the background gauge field dependence. The R-R gauge transformation rules are given by the Courant brackets between parameter vectors and gauge field vectors. There are several studies for Dp-brane on a doubled compact space [24] and on a doubled non-compact space [25].

4.1 D2-brane

The action for a D2-brane is an extension of (3.10)

$$I = I_{DBI} + I_{WZ}, \quad I_{DBI} = \int_M d^2\sigma \mathcal{L}_{DBI}$$

$$\mathcal{L}_{DBI} = -T_{D2}e^{-\phi}\sqrt{-h_F}, \quad h_F = \det h_{F\mu\nu}$$

$$\mathcal{L}_{WZ} = \frac{1}{3!}T_{D2}\epsilon^{\mu\nu\rho} \left(\partial_\mu x^m \partial_\nu x^n \partial_\rho x^l C_{mnl}^{[3]} + 2\pi\alpha' \mathcal{F}_{\mu\nu} \partial_\rho x^m C_m^{[1]} \right) .$$

The canonical momenta are defined as

$$p_m = -T_{D2}e^{-\phi}\sqrt{-h_F} \left(\frac{1}{2}h_F^{(\mu 0)}G_{mn} + \frac{1}{2}h_F^{[\mu 0]}B_{mn} \right) \partial_\mu x^n$$

$$+ T_{D2}\epsilon^{0ij} \left(\partial_i x^n \partial_j x^l \left(\frac{1}{2}C_{mnl}^{[3]} + \frac{1}{3}B_{mn}C_l^{[1]} \right) + \frac{1}{3!}2\pi\alpha' \mathcal{F}_{ij} C_m^{[1]} \right)$$

$$E^i = -2\pi\alpha' T_{D2}e^{-\phi}\sqrt{-h_F} \frac{1}{2}h_F^{[i0]} + T_{D2}\frac{1}{3}\epsilon^{0ij}2\pi\alpha' \partial_j x^m C_m^{[1]}, \quad i=1,2 . \quad (4.1)$$

The Hamiltonian and the σ^i -diffeomorphism constraints are given by [19]

$$\begin{cases} \mathcal{H}_\perp = \frac{1}{2T_{D2}}e^\phi \left(\tilde{p}_m G^{mn} \tilde{p}_n + \frac{1}{(2\pi\alpha')^2} \tilde{E}^i h_{ij} \tilde{E}^j + T_{D2}^2 e^{-2\phi} \det h_{Fij} \right) = 0 \\ \mathcal{H}_i = \partial_i x^m \tilde{p}_m + \mathcal{F}_{ij} \tilde{E}^j = \partial_i x^m p_m + F_{ij} E^j = 0 \\ \Phi = \partial_i E^i = 0 \end{cases}$$

with

$$\begin{aligned} \tilde{p}_m &\equiv p_m - B_{mn} \frac{1}{2\pi\alpha'} E^i \partial_i x^n \\ &\quad - T_{D2} \epsilon^{0ij} \left(\partial_i x^n \partial_j x^l \left(\frac{1}{2} C_{mn}^{[3]} + \frac{1}{3} B_{mn} C_l^{[1]} \right) + \frac{1}{3!} 2\pi\alpha' \mathcal{F}_{ij} C_m^{[1]} \right) \\ &= -T_{D2} e^{-\phi} \sqrt{-h_F} \frac{1}{2} h_F^{(\mu 0)} G_{mn} \partial_\mu^n \\ \tilde{E}^i &\equiv E^i - T_{D2} \frac{1}{3} \epsilon^{0ij} 2\pi\alpha' \partial_j x^m C_m^{[1]} = -2\pi\alpha' T_{D2} e^{-\phi} \sqrt{-h_F} \frac{1}{2} h_F^{[i0]} . \end{aligned} \quad (4.2)$$

The determinant term can be rewritten as

$$\det h_{Fij} = \frac{1}{2} (\epsilon^{ij} \partial_i x^m \partial_j x^n) G_{mm'} G_{nn'} (\epsilon^{i'j'} \partial_{i'} x^{m'} \partial_{j'} x^{n'}) + (2\pi\alpha' \mathcal{F}_{12})^2 . \quad (4.3)$$

Therefore the Hamiltonian for D2-brane is given by

$$\begin{aligned} \mathcal{H}_\perp &= \frac{1}{2T_{D2}} Z_M^T \mathcal{M}^{MN} Z_N \\ Z_N &= \begin{pmatrix} p_n \\ \frac{1}{2\pi\alpha'} E^i \partial_i x^n \\ T_{D2} (2\pi\alpha') \epsilon^{ij} F_{ij} \\ T_{D2} \epsilon^{ij} \partial_i x^n \partial_j x^l \end{pmatrix} \\ \mathcal{M}^{MN} &= (\mathcal{N}^T)^M_L \mathcal{M}_0^{LK} \mathcal{N}_K^N \quad (4.4) \\ \mathcal{N}_K^N &= \begin{pmatrix} \delta_k^n & -B_{kn} & -C_k^{[1]} & -C_{kn}^{[2]} \\ 0 & \delta_n^k & 0 & -C_{[n}^{[1]} \delta_{l]}^k \\ 0 & 0 & 1 & B_{nl} \\ 0 & 0 & 0 & \delta_n^k \delta_l^m \end{pmatrix} \\ \mathcal{M}_0^{LK} &= \begin{pmatrix} e^\phi G^{lk} & 0 \\ 0 & e^\phi G_{lk} \end{pmatrix} \quad . \end{aligned}$$

The Z_M basis is separated into the NS-NS sector and the R-R sector by the middle line. The upper-left part of the \mathcal{M} matrix with $\mathcal{M} = \mathcal{N} \mathcal{M}_0 \mathcal{N}$ is the same as the one for the F-string

(3.3). The parallel direction diffeomorphism constraints \mathcal{H}_i is rewritten in terms of Z_M basis by contracting with E^i and $\epsilon^{ij}\partial_j x$

$$\begin{aligned} \tilde{E}^i \mathcal{H}_i &= \epsilon^{ij} \partial_i x^m \mathcal{H}_j = 0 \\ \Rightarrow Z_M \tilde{\rho}^{MN} Z_N &= 0 \quad , \quad \tilde{\rho}^{MN} = \left(\begin{array}{cc|cc} 0 & a\delta_n^m & 0 & b_{[n}\delta_{l]}^m \\ a\delta_m^n & 0 & -b_m & 0 \\ \hline 0 & -b_n & & \\ b_{[m}\delta_{l]}^n & 0 & & \end{array} \right) \end{aligned} \quad (4.5)$$

where a, b_m are arbitrary coefficients.

The Z_M algebra is given by

$$\rho_{MN} = \left(\begin{array}{cc|cc} 0 & \frac{1}{2\pi\alpha'} E^i \delta_m^n & 0 & T_{D2} \epsilon^{ij} \partial_j x^{[n} \delta_m^{l]} \\ \frac{1}{2\pi\alpha'} E^i \delta_n^m & 0 & 2T_{D2} \epsilon^{ij} \partial_j x^m & 0 \\ \hline 0 & 2T_{D2} \epsilon^{ij} \partial_j x^n & & \\ T_{D2} \epsilon^{ij} \partial_j x^{[m} \delta_n^{l]} & 0 & & \end{array} \right) \quad (4.6)$$

where ρ_{MN} is the worldvolume vector. From the similar analysis in terms of the following notation

$$\hat{\Lambda} = \lambda + \lambda^* + \lambda^{[0]} + \lambda^{[2]} \in T \oplus T^* \oplus \Lambda^0 T^* \oplus \Lambda^2 T^*$$

we get the following extension of the Courant bracket to the one for a D2-brane

$$\begin{aligned} [\hat{\Lambda}_1, \hat{\Lambda}_2]_{D2} &= [\lambda_1, \lambda_2] + \mathcal{L}_{\lambda_{[1}} \lambda_{2]}^* + \mathcal{L}_{\lambda_{[1}} \lambda_{2]}^{[0]} + \mathcal{L}_{\lambda_{[1}} \lambda_{2]}^{[2]} - \frac{1}{2} d \left(\iota_{\lambda_{[1}} \lambda_{2]}^* + \iota_{\lambda_{[1}} \lambda_{2]}^{[0]} + \iota_{\lambda_{[1}} \lambda_{2]}^{[2]} \right) \\ &\quad + \frac{1}{2} \left(\lambda_{[1}^* \wedge \lambda_{2]}^{[0]} - \lambda_{[1}^{[0]} d \lambda_{2]}^* \right) \\ &\quad \left\{ \begin{array}{lcl} \mathcal{L}_{\lambda_1} \lambda_2^{[0]} & = & \Lambda_1^m \partial_m \Lambda_2^{[0]} T_{D2} (2\pi\alpha') \epsilon^{ij} F_{ij} \\ \mathcal{L}_{\lambda_1} \lambda_2^{[2]} & = & \left(\Lambda_1^m \partial_m \Lambda_{2;nl}^{[2]} + \partial_{[n} \Lambda_1^m \Lambda_{2;m]l} \right) T_{D2} \epsilon^{ij} \partial_i x^n \partial_j x^l \\ d(\iota_{\lambda_1} \lambda_2^{[0]}) & = & \partial_m (\Lambda_1^m \Lambda_2^{[0]}) T_{D2} (2\pi\alpha') \epsilon^{ij} F_{ij} \\ d(\iota_{\lambda_1} \lambda_2^{[2]}) & = & \partial_l (\Lambda_1^m \Lambda_{2;mn}^{[2]}) T_{D2} \epsilon^{ij} \partial_i x^n \partial_j x^l \\ \lambda_1^* \wedge d\lambda_2^{[0]} & = & \Lambda_{1[n} \partial_{l]} \Lambda_2^{[0]} T_{D2} \epsilon^{ij} \partial_i x^n \partial_j x^l \end{array} \right. \end{aligned} \quad (4.7)$$

for $K = 0$, and

$$\begin{aligned} [\hat{\Lambda}_1, \hat{\Lambda}_2]_{D2} &= [\lambda_1, \lambda_2] + \mathcal{L}_{\lambda_1} \lambda_2^* + \mathcal{L}_{\lambda_1} \lambda_2^{[0]} + \mathcal{L}_{\lambda_1} \lambda_2^{[2]} - \iota_{\lambda_2} d\lambda_1^* - \iota_{\lambda_2} d\lambda_1^{[0]} - \iota_{\lambda_2} d\lambda_1^{[2]} \\ &\quad - d\lambda_1^{[0]} \wedge \lambda_2^* + d\lambda_1^* \wedge \lambda_2^{[0]} \\ &\quad \left\{ \begin{array}{lcl} \iota_{\lambda_2} d\lambda_1^{[0]} & = & \Lambda_2^m \partial_m \Lambda_1^{[0]} T_{D2} (2\pi\alpha') \epsilon^{ij} F_{ij} \\ \iota_{\lambda_2} d\lambda_1^{[2]} & = & \Lambda_2^m \partial_{[l} \Lambda_{1]mn}^{[2]} T_{D2} \epsilon^{ij} \partial_i x^n \partial_j x^l \end{array} \right. \end{aligned} \quad (4.8)$$

for $K = -1/2$.

The gauge transformation rule is given by the Courant bracket in (4.8) as

$$\hat{C}_a^{D2} = \begin{pmatrix} e_a{}^m \\ e_{ma} \\ e_a{}^l C_l^{[1]} \\ e_a{}^l C_{mnl}^{[3]} \end{pmatrix}, \quad \hat{\xi} = \begin{pmatrix} \xi^m \\ \xi_m \\ \xi^{[0]} \\ \xi_{mn}^{[2]} \end{pmatrix}$$

$$\delta_\xi \hat{C}_a^{D2} = [\hat{\xi}, \hat{C}_a^{D2}]_{D2} \Rightarrow \begin{cases} \delta_\xi C^{[1]} = \mathcal{L}_\xi C^{[1]} + d\xi^{[0]} \\ \delta_\xi C^{[3]} = \mathcal{L}_\xi C^{[3]} + C^{[1]} \wedge d\xi + d\xi^{[2]} + B \wedge d\xi^{[0]} \end{cases} \quad (4.9)$$

where $\xi = \xi_m$ is a gauge parameter for B .

4.2 Dp-brane

The action for a Dp-brane is similar to (3.10) replacing the WZ term by

$$\begin{aligned} I &= I_{DBI} + I_{WZ}, \quad I_{DBI} = \int_M d^p \sigma \mathcal{L}_{DBI} \\ \mathcal{L}_{DBI} &= -T_{Dp} e^{-\phi} \sqrt{-h_F}, \quad h_F = \det h_{F\mu\nu} \\ I_{WZ} &= T_{Dp} \int_M e^{2\pi\alpha' \mathcal{F}} C^{RR} \\ h_{F\mu\nu} &= \partial_\mu x^m \partial_\nu x^n G_{mn} + 2\pi\alpha' \mathcal{F}_{\mu\nu} \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu, \quad \mathcal{F}_{\mu\nu} = F_{\mu\nu} + \frac{1}{2\pi\alpha'} \partial_\mu x^m \partial_\nu x^n B_{mn}. \end{aligned}$$

The canonical momenta are defined as

$$\begin{aligned} p_m &= -T_{Dp} \sqrt{-h_F} \left(\frac{1}{2} h_F^{(\mu 0)} G_{mn} + \frac{1}{2} h_F^{[\mu 0]} B_{mn} \right) \partial_\mu x^n + \frac{\partial \mathcal{L}_{WZ}}{\partial (\partial_0 x^m)} \\ E^i &= -2\pi\alpha' T_{Dp} \sqrt{-h_F} \frac{1}{2} h_F^{[i 0]} + \frac{\partial \mathcal{L}_{WZ}}{\partial F_{0i}}, \quad i=1, \dots, p. \end{aligned} \quad (4.10)$$

The Hamiltonian is given by

$$\begin{aligned} H &= \int d^p \sigma \mathcal{H} \\ \mathcal{H} &= p_m \partial_0 x^m + E^i \partial_0 A_i - \mathcal{L} \\ &= -\frac{1}{\sqrt{-h} h^{00}} \mathcal{H}_\perp - \frac{h^{0i}}{h^{00}} \mathcal{H}_i - A_0 \Phi \\ \begin{cases} \mathcal{H}_\perp = \frac{1}{2T_{Dp}} e^\phi \left(\tilde{p}_m G^{mn} \tilde{p}_n + \frac{1}{(2\pi\alpha')^2} \tilde{E}^i h_{ij} \tilde{E}^j + T_{Dp}{}^2 e^{-2\phi} \det h_{Fij} \right) = 0 \\ \mathcal{H}_i = \tilde{p}_m \partial_i x^m + \mathcal{F}_{ij} \tilde{E}^j = p_m \partial_i x^m + F_{ij} E^j = 0 \\ \Phi = \partial_i E^i = 0 \end{cases} \end{aligned}$$

with

$$\begin{aligned}
\tilde{p}_m &\equiv p_m - B_{mn} \frac{1}{2\pi\alpha'} E^i \partial_i x^n - \frac{\partial \mathcal{L}_{WZ}}{\partial(\partial_0 x^m)} \\
&= -T_{Dp} e^{-\phi} \sqrt{-h_F} \frac{1}{2} h_F^{(\mu 0)} G_{mn} \partial_\mu x^n \\
\tilde{E}^i &\equiv E^i - \frac{\partial \mathcal{L}_{WZ}}{\partial F_{0i}} = -2\pi\alpha' T_{Dp} e^{-\phi} \sqrt{-h_F} \frac{1}{2} h_F^{[i0]}.
\end{aligned} \tag{4.11}$$

4.2.1 IIA Dp-brane

Let us rewrite the Hamiltonian as the sum of bilinears. For the type IIA theory $p = 2q$ is even. \mathcal{H}_\perp for a IIA Dp-brane is written by the sum of bilinears [19] as

$$\begin{aligned}
\mathcal{H}_\perp &= \frac{1}{2T_{Dp}} Z_M^T \mathcal{M}^{MN} Z_N \\
Z_M &= \left(\begin{array}{c} p_m \\ \hline \frac{1}{2\pi\alpha'} E^i \partial_i x^m \\ T_{Dp} (2\pi\alpha')^q F^q \\ \vdots \\ T_{Dp} (2\pi\alpha') \epsilon^{i_1 \dots i_p} F_{i_1 i_2} \partial_{i_3} x^{m_1} \dots \partial_{i_p} x^{m_{p-2}} \\ T_{Dp} \epsilon^{i_1 \dots i_p} \partial_{i_1} x^{m_1} \dots \partial_{i_p} x^{m_p} \end{array} \right) \\
\mathcal{M}^{MN} &= (\mathcal{N}^T)^M{}_L \mathcal{M}_0^{LK} \mathcal{N}_K^N \\
&\quad \left(\begin{array}{c|ccccc} \delta_k^n & -B_{kn} & -C^{[1]} & \dots & -\sum_{r=0}^q C^{[p-1-2r]} B^r & -\sum_{r=0}^q C^{[p+1-2r]} B^r \\ 0 & \delta_n^k & 0 & \dots & -\sum_{r=0}^q C^{[p-3-2r]} B^r & -\sum_{r=0}^q C^{[p-1-2r]} B^r \end{array} \right) \\
\mathcal{N}_K^N &= \left(\begin{array}{cc|ccccc} 0 & 0 & 1 & \dots & B^{q-1} & & B^q \\ & & & \vdots & & & \\ 0 & 0 & 0 & \dots & \mathbf{1} & & B \\ 0 & 0 & 0 & \dots & 0 & & \mathbf{1} \end{array} \right) \\
\mathcal{M}_0^{LK} &= \left(\begin{array}{c|ccccc} e^\phi G^{lk} & & & & & \\ \hline & e^\phi G_{lk} & & & & \\ & & e^{-\phi} & & & \\ & & & \dots & & \\ & & & & e^{-\phi} G_{l_1 k_1} \dots G_{l_{p-2} k_{p-2}} & \\ & & & & & e^{-\phi} G_{l_1 k_1} \dots G_{l_p k_p} \end{array} \right)
\end{aligned}$$

where the NS-NS two-form $B_{mn} = B^{[2]}$ is denoted by B .

The worldvolume diffeomorphism constraints $\mathcal{H}_i = 0$ is written in a bilinear form by contracting with $\tilde{E}^i, \dots, \epsilon^{i_1 \dots i_p} F_{i_1 i_2} \dots \partial_{i_p} x$,

$$\begin{aligned} \tilde{E}^i \mathcal{H}_i &= \epsilon^{i_1 \dots i_{p-1} i} F_{i_1 i_2} \dots \partial_{i_{p-1}} x \mathcal{H}_i = \epsilon^{i_1 \dots i_{p-1} i} \partial_{i_1} x \dots \partial_{i_{p-1}} x \mathcal{H}_i = 0 \\ \Rightarrow Z_M \tilde{\rho}^{MN} Z_N &= 0 \\ \tilde{\rho}^{MN} &= \left(\begin{array}{cc|ccccc} 0 & a\delta_n^m & 0 & b_{[2]}^m & \dots & \dots & c_{[p]}^m \\ a\delta_m^n & 0 & b_{[0]m} & \dots & \dots & c_{[p-2]m} & 0 \\ \hline 0 & b_{[0]n} & & & & & \\ b_{[2]}^n & \cdot & & & & & \\ \vdots & \vdots & & & & & \\ \cdot & c_{[p-2]n} & & & & & \\ c_{[p]}^n & 0 & & & & & \end{array} \right) \\ b_{[2]}^m &= \beta_{[n} \delta_{l]}^m, & b_{[0]m} &= -\beta_m \\ c_{[p]}^m &= \gamma_{[n_1 \dots n_{p-1}} \delta_{n_p]}^m, & c_{[p-2]m} &= -_p C_2 \gamma_{[n_1 \dots n_{p-2}m]} \end{aligned}$$

with arbitrary coefficients a, β, γ .

The Z_M algebra is given by

$$\{Z_M(\sigma), Z_N(\sigma')\} = i \rho_{MN}^i \partial_i \delta^{(p)}(\sigma - \sigma')$$

$$\begin{aligned} \rho_{MN}^i &= \left(\begin{array}{cc|ccccc} 0 & \frac{1}{2\pi\alpha'} E^i \delta_m^n & 0 & \rho_{14} & \dots & \dots & \rho_{16} \\ \frac{1}{2\pi\alpha'} E^i \delta_n^m & 0 & \rho_{23} & \dots & \dots & \rho_{25} & 0 \\ \hline 0 & \rho_{23} & & & & & \\ \rho_{14} & \vdots & & & & & \\ \vdots & \rho_{25} & & & & & \\ \rho_{16} & 0 & & & & & \end{array} \right) \\ \rho_{14} &= T_{Dp} \epsilon^{i_1 \dots i_{p-1}} (2\pi\alpha')^{q-1} F_{i_1 \dots i_{p-2}}^{q-1} \partial_{i_{p-1}} x^{[n_1} \delta_{m]}^{n_2]} \\ \rho_{23} &= T_{Dp} \epsilon^{i_1 \dots i_{p-1}} (2\pi\alpha')^{q-1} F_{i_1 \dots i_{p-2}}^{q-1} \partial_{i_{p-1}} x^m \\ \rho_{16} &= T_{Dp} \epsilon^{i_1 \dots i_{p-1}} \partial_{i_1} x^{[n_1} \dots \partial_{i_{p-1}} x^{n_{p-1}} \delta_{m]}^{n_p]} \\ \rho_{25} &= T_{Dp} \epsilon^{i_1 \dots i_{p-1}} \partial_{i_1} x^{[n_1} \dots \partial_{i_{p-1}} x^{m]} \end{aligned}$$

where ρ_{MN}^i is a worldvolume vector. From the similar analysis we get the following extension of the Courant bracket to the one for a IIA Dp-brane

$$\hat{\Lambda} = \lambda + \lambda^* + \lambda^{[0]} + \dots + \lambda^{[p]} \in T \oplus T^* \oplus \Lambda^0 T^* \oplus \dots \oplus \Lambda^p T^*$$

$$\begin{aligned}
[\hat{\Lambda}_1, \hat{\Lambda}_2]_{Dp} &= [\lambda_1, \lambda_2] + \mathcal{L}_{\lambda_{[1}} \lambda_{2]}^* + \mathcal{L}_{\lambda_{[1}} \lambda_{2]}^{[0]} + \cdots + \mathcal{L}_{\lambda_{[1}} \lambda_{2]}^{[p]} \\
&\quad - \frac{1}{2} d(\iota_{\lambda_{[1}}} \lambda_{2]}^* + \iota_{\lambda_{[1}}} \lambda_{2]}^{[0]} + \cdots + \iota_{\lambda_{[1}}} \lambda_{2]}^{[p]}) \\
&\quad + \sum_{s=1}^q \frac{s}{(p+2-2s)!} (\lambda_{[1]}^* \wedge d\lambda_{2]}^{[p-2s]} + d\lambda_{[1]}^* \wedge \lambda_{2]}^{[p-2s]}) \quad \text{for } K=0 \quad (4.12)
\end{aligned}$$

$$\begin{aligned}
[\hat{\Lambda}_1, \hat{\Lambda}_2]_{Dp} &= [\lambda_1, \lambda_2] + \mathcal{L}_{\lambda_1} \lambda_2^* + \mathcal{L}_{\lambda_1} \lambda_2^{[0]} + \cdots + \mathcal{L}_{\lambda_1} \lambda_2^{[p]} \\
&\quad - \iota_{\lambda_2} d\lambda_1^* - \iota_{\lambda_2} d\lambda_1^{[0]} - \cdots - \iota_{\lambda_2} d\lambda_1^{[p]} \\
&\quad + \sum_{s=1}^q \frac{2s}{(p+2-2s)!} (d\lambda_1^* \wedge d\lambda_2^{[p-2s]} - d\lambda_1^{[p-2s]} \wedge \lambda_2^*) \quad \text{for } K = -\frac{1}{2} \quad (4.13)
\end{aligned}$$

The gauge transformation rule for the R-R gauge fields is given by the Courant bracket in (4.13) as

$$\begin{aligned}
\hat{C}_a^{Dp} &= \begin{pmatrix} e_a^m \\ e_{ma} \\ \hline e_a^l C_l^{[1]} \\ \vdots \\ e_a^l C_{m_1 \cdots m_p l}^{[p+1]} \end{pmatrix}, \quad \hat{\xi} = \begin{pmatrix} \xi^m \\ \xi_m \\ \hline \xi^{[0]} \\ \vdots \\ \xi_{m_1 \cdots m_p}^{[p]} \end{pmatrix} \\
\delta_{\xi} \hat{C}_a^{Dp} = [\hat{\xi}, \hat{C}_a^{Dp}]_{Dp} &\Rightarrow \begin{cases} \delta_{\xi} C^{[1]} = \mathcal{L}_{\xi} C^{[1]} + d\xi^{[0]} \\ \vdots \\ \delta_{\xi} C^{[p+1]} = \mathcal{L}_{\xi} C^{[p+1]} + C^{[p-1]} \wedge d\xi + d\xi^{[p]} + B \wedge d\xi^{[p-2]} \end{cases} \quad (4.14)
\end{aligned}$$

where $\xi = \xi_m$ is a gauge parameter for B .

4.2.2 IIB Dp-brane

For the type IIB $p = 2q + 1$ is odd. Then \mathcal{H}_{\perp} for a IIB Dp-brane is written by the sum of bilinears [18] as

$$\begin{aligned}
\mathcal{H}_{\perp} &= \frac{1}{2T_{Dp}} Z_M^T \mathcal{M}^{MN} Z_N \\
Z_M &= \begin{pmatrix} p_m \\ \hline \frac{1}{2\pi\alpha'} E^i \partial_i x^m \\ T_{Dp} (2\pi\alpha')^q F^q \wedge dx^m \\ \vdots \\ T_{Dp} (2\pi\alpha') \epsilon^{i_1 \cdots i_p} F_{i_1 i_2} \partial_{i_3} x^{m_1} \cdots \partial_{i_p} x^{m_{p-2}} \\ T_{Dp} \epsilon^{i_1 \cdots i_p} \partial_{i_1} x^{m_1} \cdots \partial_{i_p} x^{m_p} \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}^{MN} &= (\mathcal{N}^T)^M{}_L \mathcal{M}_0{}^{LK} \mathcal{N}_K{}^N \\
\mathcal{N}_K{}^N &= \left(\begin{array}{cc|ccccc} \delta_k{}^n & -B_{kn} & -C^{[2]} - C^{[0]}B & \dots & -\sum_{r=0}^q C^{[p-1-2r]}B^r & -\sum_{r=0}^q C^{[p+1-2r]}B^r \\ 0 & \delta^k{}_n & C^{[0]} & \dots & -\sum_{r=0}^q C^{[p-3-2r]}B^r & -\sum_{r=0}^q C^{[p-1-2r]}B^r \end{array} \right) \\
\mathcal{M}_0^{LK} &= \left(\begin{array}{cc|ccccc} e^\phi G^{lk} & & & & & & \\ e^\phi G_{lk} & & & & & & \\ \hline & & e^{-\phi} G_{lk} & & & & \\ & & & \dots & & & \\ & & & & e^{-\phi} G_{l_1 k_1} \cdots G_{l_{p-2} k_{p-2}} & & \\ & & & & & e^{-\phi} G_{l_1 k_1} \cdots G_{l_p k_p} & \end{array} \right) \tag{4.15}
\end{aligned}$$

The worldvolume diffeomorphism constraints $\mathcal{H}_i = 0$ is written in a bilinear form by contracting with $E^i, \dots, \epsilon^{i_1 \dots i_p} F_{i_1 i_2} \dots \partial_{i_p} x$,

$$\begin{aligned}
\tilde{E}^i \mathcal{H}_i &= \epsilon^{i_1 \dots i_{p-1}} \partial_{i_1} x \cdots \partial_{i_{p-1}} x \mathcal{H}_i = \epsilon^{i_1 \dots i_{p-1}} F_{i_1 i_2} \cdots \partial_{i_{p-1}} x \mathcal{H}_i = 0 \\
Z_M \tilde{\rho}^{MN} Z_N &= 0 \\
\tilde{\rho}^{MN} &= \left(\begin{array}{cc|cc} 0 & a\delta_n^m & b\delta_n^m & \dots & c_{[p]}^m \\ a\delta_m^n & 0 & \dots & c_{[p-2]m} & 0 \\ \hline b\delta_m^n & \vdots & & & \\ \vdots & c_{[p-2]n} & & & \\ c_{[p]}^n & 0 & & & \end{array} \right) \\
c_{[p]}^m &= \gamma_{[n_1 \dots n_{p-1}} \delta_{n_p]}^m, \quad c_{[p-2]m} = -_p C_2 \gamma_{[n_1 \dots n_{p-2}m]}
\end{aligned}$$

with arbitrary coefficients a, b, γ .

The Z_M algebra is given by

$$\{Z_M(\sigma), Z_N(\sigma')\} = i\rho_{MN}^i \partial_i \delta^{(p)}(\sigma - \sigma')$$

$$\rho_{MN}^i = \left(\begin{array}{cc|ccc} 0 & \frac{1}{2\pi\alpha'} E^i \delta_m^n & \rho_{13} & \cdots & \rho_{15} \\ \frac{1}{2\pi\alpha'} E^i \delta_n^m & 0 & \cdots & \rho_{24} & 0 \\ \hline \rho_{13} & \vdots & & & \\ \vdots & \rho_{24} & & & \\ \rho_{15} & 0 & & & \end{array} \right)$$

$$\rho_{13} = T_{Dp} \epsilon^{ii_1 \cdots i_{p-1}} (2\pi\alpha')^q (F^q)_{i_1 \cdots i_{p-2}} \delta_m^n$$

$$\rho_{15} = T_{Dp} \epsilon^{ii_1 \cdots i_{p-1}} \partial_{i_1} x^{[n_1} \cdots \partial_{i_{p-1}} x^{n_{p-1}] \delta_m^{n_p]}$$

$$\rho_{24} = T_{Dp} \epsilon^{ii_1 \cdots i_{p-1}} \partial_{i_1} x^{[m} \cdots \partial_{i_{p-1}} x^{n_{p-2}]}$$

where ρ_{MN}^i is a worldvolume vector. The Courant bracket to the one for a IIB Dp-brane in terms of a vector as

$$\hat{\Lambda} = \lambda + \lambda^* + \lambda^{[1]} + \cdots + \lambda^{[p]} \in T \oplus T^* \oplus \Lambda^1 T^* \oplus \cdots \oplus \Lambda^p T^*$$

$$\begin{aligned} [\hat{\Lambda}_1, \hat{\Lambda}_2]_{Dp} &= [\lambda_1, \lambda_2] + \mathcal{L}_{\lambda_{[1}}} \lambda_{[2]}^* + \mathcal{L}_{\lambda_{[1]}} \lambda_{[2]}^{[1]} + \cdots + \mathcal{L}_{\lambda_{[1]}} \lambda_{[2]}^{[p]} \\ &\quad - \frac{1}{2} d(\iota_{\lambda_{[1]}} \lambda_{[2]}^* + \iota_{\lambda_{[1]}} \lambda_{[2]}^{[1]} + \cdots + \iota_{\lambda_{[1]}} \lambda_{[2]}^{[p]}) \\ &\quad + \sum_{s=1}^q \frac{s}{(p+2-2s)!} (\lambda_{[1]}^* \wedge d\lambda_{[2]}^{[p-2s]} + d\lambda_{[1]}^* \wedge \lambda_{[2]}^{[p-2s]}) \quad \text{for } K=0 \end{aligned} \quad (4.16)$$

$$\begin{aligned} [\hat{\Lambda}_1, \hat{\Lambda}_2]_{Dp} &= [\lambda_1, \lambda_2] + \mathcal{L}_{\lambda_1} \lambda_2^* + \mathcal{L}_{\lambda_1} \lambda_2^{[1]} + \cdots + \mathcal{L}_{\lambda_1} \lambda_2^{[p]} \\ &\quad - \iota_{\lambda_2} d\lambda_1^* - \iota_{\lambda_2} d\lambda_1^{[1]} - \cdots - \iota_{\lambda_2} d\lambda_1^{[p]} \\ &\quad + \sum_{s=1}^q \frac{2s}{(p+2-2s)!} (d\lambda_1^* \wedge \lambda_2^{[p-2s]} - d\lambda_1^{[p-2s]} \wedge \lambda_2^*) \quad \text{for } K = -\frac{1}{2} \end{aligned} \quad (4.17)$$

The gauge transformation rule for the R-R gauge fields is given by the Courant bracket in (4.17) as

$$\hat{C}_a^{Dp} = \begin{pmatrix} e_a^m \\ e_{ma} \\ \hline e_a^l C_{ml}^{[2]} \\ \vdots \\ e_a^l C_{m_1 \cdots m_p l}^{[p+1]} \end{pmatrix}, \quad \hat{\xi} = \begin{pmatrix} \xi^m \\ \xi_m \\ \hline \xi_m^{[1]} \\ \vdots \\ \xi_{m_1 \cdots m_p}^{[p]} \end{pmatrix}$$

$$\delta_\xi \hat{C}_a^{Dp} = [\hat{\xi}, \hat{C}_a^{Dp}]_{Dp} \Rightarrow \begin{cases} \delta_\xi C^{[2]} = \mathcal{L}_\xi C^{[2]} + d\xi^{[1]} \\ \vdots \\ \delta_\xi C^{[p+1]} = \mathcal{L}_\xi C^{[p+1]} - C^{[p-1]} \wedge d\xi + d\xi^{[p]} + B \wedge d\xi^{[p-2]} \end{cases} \quad (4.18)$$

where $\xi = \xi_m$ is a gauge parameter for B .

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Appendix

A Determinant

In this appendix we show that the $\det h_{Fij}$ is written in a bilinear form, where $h_{Fij} = h_{ij} + f_{ij}$ contains a symmetric matrix $h_{ij} = \partial_i x^m \partial_j x^n G_{mn}$ and an antisymmetric matrix $f_{ij} = 2\pi\alpha' F_{ij} + \partial_i x^m \partial_j x^n B_{mn}$. The coefficients of each term are determined.

As a nontrivial example we begin by a determinant $\det h_{Fij}$ for $p = 4$ case with $i, j = 1, \dots, 4$;

$$\begin{aligned}
\det h_{Fij} &= \frac{1}{4!} \epsilon^{ijkl} h_{ii'} h_{jj'} h_{kk'} h_{ll'} \epsilon^{i'j'k'l'} \\
&\quad + \frac{4C_2}{4!} \epsilon^{ijkl} h_{ii'} h_{jj'} f_{kk'} f_{ll'} \epsilon^{i'j'k'l'} \\
&\quad + \frac{1}{4!} \epsilon^{ijkl} f_{ii'} f_{jj'} f_{kk'} f_{ll'} \epsilon^{i'j'k'l'} \\
&= \frac{1}{4!} \epsilon^{ijkl} \partial_i x^{m_1} \partial_i x^{m_2} \partial_i x^{m_3} \partial_i x^{m_4} G_{m_1 n_1} G_{m_2 n_2} G_{m_3 n_3} G_{m_4 n_4} \epsilon^{i'j'k'l'} \partial_{i'} x^{n_1} \partial_{j'} x^{n_2} \partial_{k'} x^{n_3} \partial_{l'} x^{n_4} \\
&\quad + \frac{4C_2}{4! \cdot 2} \epsilon^{ijkl} \partial_i x^{m_1} \partial_j x^{m_2} f_{kl} G_{m_1 n_1} G_{m_2 n_2} \epsilon^{i'j'k'l'} \partial_{i'} x^{n_1} \partial_{j'} x^{n_2} f_{k'l'} \\
&\quad + \frac{3}{4! \cdot 2 \cdot 4} \epsilon^{ijkl} f_{ij} f_{kl} \epsilon^{i'j'k'l'} \partial_{i'} x^{n_1} f_{i'j'} f_{k'l'} \quad .
\end{aligned}$$

Useful relation obtained by taking totally antisymmetric indices for five indices is the following:

$$\begin{aligned}
0 &= \epsilon^{ijkl} \epsilon^{i'j'k'l'} (h_{ii'} h_{jj'} f_{kk'} f_{ll'} + 4\text{terms totally antisymmetric in } ijkll') \\
&= \epsilon^{ijkl} \epsilon^{i'j'k'l'} (2h_{ii'} h_{jj'} f_{kk'} f_{ll'} - h_{kk'} h_{ll'} f_{i'j'} f_{ij}) \\
\Rightarrow & \epsilon^{ijkl} \epsilon^{i'j'k'l'} h_{ii'} h_{jj'} f_{kk'} f_{ll'} \\
&= \frac{1}{2} \left(\epsilon^{ijkl} \partial_i x^{m_1} \partial_j x^{m_2} f_{kl} \right) G_{m_1 n_1} G_{m_2 n_2} \left(\epsilon^{i'j'k'l'} \partial_{i'} x^{n_1} \partial_{j'} x^{n_2} f_{k'l'} \right) \quad .
\end{aligned} \tag{A.1}$$

Analogously the factorization of each term can be confirmed.

Systematic analysis of generic dimension is given as follows. The generic form of a Dp-brane in type IIA: ($p = 2q$)

$$\begin{aligned}
\det h_{Fij} &= \frac{1}{p!} G_{m_1 n_1} \cdots G_{m_p n_p} \left(\epsilon^{i_1 \cdots i_p} \partial_{i_1} x^{m_1} \cdots \partial_{i_p} x^{m_p} \right)^2 \\
&\quad + \frac{1}{(2^1)^2} G_{m_1 m'_1} \cdots G_{m_{p-2} m'_{p-2}} \left(\epsilon^{i_1 \cdots i_p} \partial_{i_1} x^{m_1} \cdots \partial_{i_{p-2}} x^{m_{p-2}} f_{i_{p-1} i_p} \right)^2 \\
&\quad + \frac{1}{(2^3)^2} G_{m_1 m'_1} \cdots G_{m_{p-4} m'_{p-4}} \left(\epsilon^{i_1 \cdots i_p} \partial_{i_1} x^{m_1} \cdots \partial_{i_{p-4}} x^{m_{p-4}} f_{i_{p-3} i_{p-2}} f_{i_{p-1} i_p} \right)^2 \\
&\quad + \dots \\
&\quad + \frac{1}{(2^{\alpha_q})^2} \left(\epsilon^{i_1 \cdots i_p} f_{i_1 i_2} \cdots f_{i_{p-1} i_p} \right)^2 \\
&= \frac{1}{p!} G_{m_1 n_1} \cdots G_{m_p n_p} \left(\epsilon^{i_1 \cdots i_p} \partial_{i_1} x^{m_1} \cdots \partial_{i_p} x^{m_p} \right)^2 \\
&\quad + \sum_{k=1}^q \frac{1}{(2^{\alpha_k})^2} G_{m_{2k+1} m'_{2k+1}} \cdots G_{m_p m'_p} \left[\epsilon^{i_1 \cdots i_p} (f_{i_1 i_2} \cdots f_{i_{2k-1} i_{2k}}) \partial_{i_{2k+1}} x^{m_{2k+1}} \cdots \partial_{i_p} x^{m_p} \right]^2, \\
\alpha_k &= {}_{2k}C_2 \times {}_{2k-2}C_2 \times \cdots \times {}_2C_2 \times \frac{1}{k!} = \frac{(2k)!}{(2!)^k k!}, \\
\alpha_q &= {}_{2q}C_2 \times {}_{2q-2}C_2 \times \cdots \times {}_2C_2 \times \frac{1}{q!} = \frac{(2q)!}{(2!)^q q!}.
\end{aligned} \tag{A.2}$$

The generic form of a Dp-brane in type IIB: ($p = 2q + 1$)

$$\begin{aligned}
\det h_{Fij} &= \frac{1}{p!} G_{m_1 n_1} \cdots G_{m_p n_p} \left(\epsilon^{i_1 \cdots i_p} \partial_{i_1} x^{m_1} \cdots \partial_{i_p} x^{m_p} \right)^2 \\
&\quad + \frac{1}{(2^1)^2} G_{m_1 m'_1} \cdots G_{m_{p-2} m'_{p-2}} \left(\epsilon^{i_1 \cdots i_p} \partial_{i_1} x^{m_1} \cdots \partial_{i_{p-2}} x^{m_{p-2}} f_{i_{p-1} i_p} \right)^2 \\
&\quad + \frac{1}{(2^3)^2} G_{m_1 m'_1} \cdots G_{m_{p-4} m'_{p-4}} \left(\epsilon^{i_1 \cdots i_p} \partial_{i_1} x^{m_1} \cdots \partial_{i_{p-4}} x^{m_{p-4}} f_{i_{p-3} i_{p-2}} f_{i_{p-1} i_p} \right)^2 \\
&\quad + \dots \\
&\quad + \frac{1}{(2^{\alpha_q})^2} G_{mm'} \left(\epsilon^{i_1 \cdots i_p} f_{i_1 i_2} \cdots f_{i_{2q-1} i_{2q}} \partial_{i_{2q+1}} x^m \right)^2 \\
&= \frac{1}{p!} G_{m_1 n_1} \cdots G_{m_p n_p} \left(\epsilon^{i_1 \cdots i_p} \partial_{i_1} x^{m_1} \cdots \partial_{i_p} x^{m_p} \right)^2 \\
&\quad + \sum_{k=1}^q \frac{1}{(2^{\alpha_k})^2} G_{m_{2k+1} m'_{2k+1}} \cdots G_{m_p m'_p} \left[\epsilon^{i_1 \cdots i_p} (f_{i_1 i_2} \cdots f_{i_{2k-1} i_{2k}}) \partial_{i_{2k+1}} x^{m_{2k+1}} \cdots \partial_{i_p} x^{m_p} \right]^2, \\
\alpha_k &= {}_{2k}C_2 \times {}_{2k-2}C_2 \times \cdots \times {}_2C_2 \times \frac{1}{k!} = \frac{(2k)!}{(2!)^k k!}.
\end{aligned} \tag{A.3}$$

Formally the determinant in type IIB has the same representation as to the one in type IIA.

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